

ON THE SPARING NUMBER OF THE EDGE-CORONA OF GRAPHS

K. P. CHITHRA

*Naduvath Mana, Nandikkara
Thrissur-680301, Kerala, India.
E-mail: chithrasudev@gmail.com*

K. A. GERMINA

*PG & Research Department of Mathematics
Mary Matha Arts & Science College
Mananthavady, Wayanad-670645, Kerala, India.
E-mail: srgerminaka@gmail.com*

N.K. SUDEV

*Department of Mathematics
Vidya Academy of Science & Technology
Thalakkottukara, Thrissur - 680501, Kerala, India.
E-mail: sudevnk@gmail.com*

Abstract

Let \mathbb{N}_0 be the set of all non-negative integers and $\mathcal{P}(\mathbb{N}_0)$ be its the power set. An integer additive set-indexer (IASI) of a graph G is an injective function $f : V(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$ such that the induced function $f^+ : E(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is also injective, where $f(u) + f(v)$ is the sum set of $f(u)$ and $f(v)$. An integer additive set-indexer f is said to be a weak integer additive set-indexer (weak IASI) if $|f^+(uv)| = \max(|f(u)|, |f(v)|) \forall uv \in E(G)$. The minimum number of singleton set-labeled edges required for the graph G to admit an IASI is called the sparing number of the graph. In this paper, we discuss the admissibility of weak IASI by a particular type of graph product called the edge corona of two given graphs and determine the sparing number of the edge corona of certain graphs.

Key Words: Integer additive set-indexers, mono-indexed elements of a graphs, weak integer additive set-indexers, sparing number of a graph, edge corona of a graph.

AMS Subject Classification: 05C78

1 Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [1], [9] and [17]. Unless mentioned otherwise, all graphs considered here are simple, finite and have no isolated vertices.

The sum set of two sets A and B , denoted $A + B$, is the set defined by $A + B = \{a + b : a \in A, b \in B\}$. If either A or B is countably infinite, then their sum set will also be countably infinite. Hence, all sets we consider in this study are finite sets. The cardinality of a set A is denoted by $|A|$. The power set of a set A is denoted by $\mathcal{P}(A)$.

Let \mathbb{N}_0 denote the set of all non-negative integers. An *integer additive set-indexer* (IASI, in short) of a graph G is defined in [6] as an injective function $f : V(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$ such that the induced function $f^+ : E(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is also injective.

A *weak IASI* f is (see [7]) an IASI such that $|f^+(uv)| = \max(|f(u)|, |f(v)|)$ for all $u, v \in V(G)$. A weak IASI f is said to be *weakly uniform IASI* if $|f^+(uv)| = k$, for all $u, v \in V(G)$ and for some positive integer k . A graph which admits a weak IASI may be called a *weak IASI graph*.

The following result is a necessary and sufficient condition for a given graph to admit a weak IASI.

Lemma 1.1. [7] *A graph G admits a weak integer additive set-indexer if and only if every edge of G has at least one mono-indexed end vertex.*

The following definitions are made in [11]. The cardinality of the labeling set of an element (vertex or edge) of a graph G is called the *set-indexing number* of that element. An element (a vertex or an edge) of graph which has the set-indexing number 1 is called a *mono-indexed element* of that graph. The *sparing number* of a graph G is defined to be the minimum number of mono-indexed edges required for G to admit a weak IASI and is denoted by $\varphi(G)$.

Certain Studies about weak IASIs have been done already and the following are some major results about weak IASI graphs relevant in this study.

Theorem 1.2. [11] *A subgraph of weak IASI graph is also a weak IASI graph.*

Theorem 1.3. [11] *A graph G admits a weak IASI if and only if G is bipartite or it has at least one mono-indexed edge. Also, the sparing number of a bipartite graph G is 0.*

Theorem 1.4. [11] *Let C_n be a cycle of length n which admits a weak IASI, for a positive integer n . Then, C_n has an odd number of mono-indexed edges when it is an odd cycle and has even number of mono-indexed edges, when it is an even cycle. An odd cycle C_n has a weak IASI if and only if it has at least one mono-indexed edge.*

Theorem 1.5. [12] *The graph $G_1 \cup G_2$ admits a weak IASI if and only if both G_1 and G_2 are weak IASI graphs. More over, $\varphi(G_1 \cup G_2) = \varphi(G_1) + \varphi(G_2) - \varphi(G_1 \cap G_2)$.*

Theorem 1.6. [11] *A complete graph can have at most one vertex that is not mono-indexed. Also, the sparing number of a complete graph K_n is $\frac{1}{2}(n-1)(n-2)$.*

The admissibility of weak IASI by certain graph products and their sparing numbers have been studied in [2], [3] and [15]. In this paper, our intention is to study about the admissibility of weak IASI by a particular product, called edge corona, of two given graphs and estimate the corresponding sparing number.

2 The Sparing Number of Edge Corona of Graphs

Let us first recall the definition of the edge corona of two graphs.

Definition 2.1. [10] Let G_1 be a graph with n_1 vertices and m_1 edges and G_2 be a graph with n_2 vertices and m_2 edges. Then, the *edge corona* of G_1 and G_2 , denoted by $G_1 \diamond G_2$, is the graph obtained by taking m_1 copies of G_2 and then joining the end vertices of i -th edge of G_1 to every vertex in the i -th copy of G_2 .

Figure 1 is an example for the graph which is the edge corona of the cycles C_5 and C_3 .

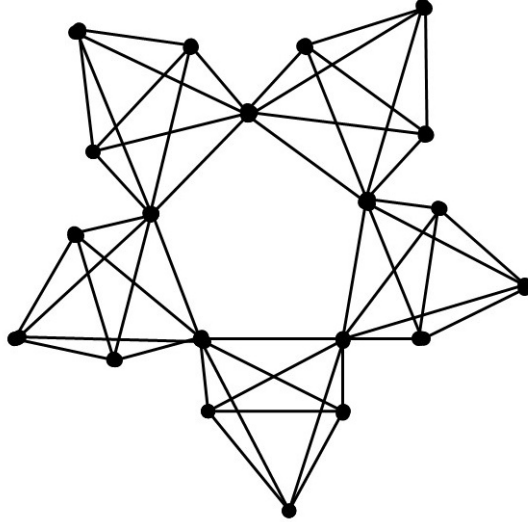


Figure 1: The edge corona $C_5 \diamond C_3$.

The weak IASIs of G_1 and G_2 may not induce a weak IASI for $G_1 \diamond G_2$. Hence, we need to define an IASI independently for a graph product.

We say that a graph G is said to be a *1-uniform graph* if the set-labels of all elements (vertices and edges) of G are singleton sets. By the term an *integral multiple of a set A* , we mean the set obtained by multiplying every element of A by a same integer.

The following theorem establishes a necessary condition for the edge corona of two weak IASI graphs to admit a weak IASI.

Theorem 2.2. *For two given graphs G_1 and G_2 , if $G_1 \diamond G_2$ admits a weak IASI, then either G_1 is 1-uniform or $m_1 - m'_1$ copies of G_2 are 1-uniform, where m'_1 is the number of mono-indexed edges in G_1 .*

Proof. Let G_1 be a graph on n_1 vertices and m_1 edges and G_2 be a graph on n_2 vertices and m_2 edges. Let $V(G_1) = \{v_1, v_2, \dots, v_{n_1}\}$ and $V(G_2) = \{u_1, u_2, \dots, u_{n_2}\}$ be the vertex sets and $E(G_1) = \{e_1, e_2, \dots, e_{m_1}\}$ and $E(G_2) = \{e'_1, e'_2, \dots, e'_{m_2}\}$ be the edge sets of G_1 and G_2 respectively. Let $G_{2,j}$ be the j -th copy of G_2 corresponding to the j -th edge $e_j = v_r v_s$ of G_1 in $G_1 \diamond G_2$ and $V(G_{2,j}) = \{u_{1j}, u_{2j}, \dots, u_{n_2j}\}$. Then, the subgraph of $G_1 \diamond G_2$ induced by the vertices $\{v_r, v_s, u_{kj}, u_{lj}\}$ is the complete graph K_4 , for any two adjacent vertices u_{kj} and u_{lj} in $G_{2,j}$. That is, all edges of $G_{2,i}$ are the edges of different complete graphs K_4 in $G_1 \diamond G_2$, all of these complete graphs have the common edge $e_j = v_r v_s$.

First assume that $G_1 \diamond G_2$ admits a weak IASI. Then, we have to consider the following two cases.

Case-1: Assume that G_1 is not 1-uniform. Then, G_1 will have some elements which are not mono-indexed. Without loss of generality, assume that the edge e_j is not mono-indexed. Then either v_r or v_s must have a non-singleton set-label. Let v_r be the vertex that is not mono-indexed. Then, by Theorem 1.6, no other vertex v_{lj} can have a non-singleton set-label. Therefore, the copy $G_{2,j}$ is 1-uniform. This argument is valid for the copies of G_2 corresponding to all edges of G that are not mono-indexed. Therefore, at least $m_1 - m'_1$ copies of G_2 must be 1-uniform, where m'_1 is the number of mono-indexed edges in G_1 .

Case-2: Assume that no copy of G_2 is 1-uniform. Then, each copy $G_{2,j}$ of G_2 has at least one edge that is not mono-indexed. Let the edge $u_{kj} u_{lj}$ of $G_{2,j}$ has the non-singleton set-label. Then, by Theorem 1.6, the end vertices v_r and v_s of the corresponding edge e_j of G_1 can not have non-singleton set-label. Hence, as no copy of G_2 are 1-uniform, no vertex of G_1 can have a non-singleton set-label. That is, G_1 is 1-uniform. \square

The converse of the theorem is also valid for with respect to the weak IASIs defined on G_1 and G_2 . Let f_1 and f_2 the weak IASIs defined on G_1 and G_2 , which need not be 1-uniform. The vertices of the copies of G_2 corresponding to the non-mono-indexed edges of G_1 need to be re-labeled using distinct singleton sets and the vertices of the copies of G_2 corresponding to the mono-indexed edges of G_1 can be labeled by distinct integral multiples of the set-labels of the corresponding vertices of G_2 . Clearly, this new labeling will be a weak IASI of $G_1 \diamond G_2$. Hence, we have the following necessary and sufficient condition for the edge corona of two weak IASI graphs to admit a weak IASI.

Theorem 2.3. *For given weak IASI graphs G_1 and G_2 , $G_1 \diamond G_2$ admits a weak IASI if and only if $m_1 - m'_1$ copies of G_2 are 1-uniform, where m'_1 is the number of mono-indexed edges in G_1 .*

In view of Theorem 2.3, we can estimate the number of mono-indexed edges in the edge corona of two given graphs.

Theorem 2.4. For given graphs G_1 and G_2 , the number of mono-indexed edges in $G_1 \diamond G_2$ is $m'_1(1 + m'_2 + 2n'_2) + (m_1 - m'_1)(m_2 + n_2)$, where m_i is the number of edges and n_i is the number of vertices of G_i for $i = 1, 2$ and m'_i is the number of mono-indexed edges and n'_i is the number of mono-indexed vertices of G_i with respect to a weak IASI defined on G_i for $i = 1, 2$.

Proof. Let G_1 be a graph on n_1 vertices and m_1 edges and G_2 be a graph on n_2 vertices and m_2 edges. Let f_1 and f_2 be the weak IASIs defined on G_1 and G_2 respectively. Let n'_i and m'_i be the number of vertices and edges of G_i that are mono-indexed under the weak IASI f_i for $i = 1, 2$.

Let $G = G_1 \diamond G_2$ be a weak IASI graph. Assume that G_1 is not 1-uniform. Then, G_1 has some elements having non-singleton set-labels. Then, by Theorem 2.2, $m_1 - m'_1$ copies of G_2 must be 1-uniform. Let \mathfrak{C}_1 be the set of all 1-uniform copies of G_2 in $G_1 \diamond G_2$. Therefore, the members of \mathfrak{C}_1 contributes a total of $(m_1 - m'_1)m_2$ mono-indexed edges to $G_1 \diamond G_2$.

In the remaining m'_1 copies of G_2 , we can label the vertices by the distinct integral multiples of the set-labels of the corresponding vertices of G_2 with respect to f_2 . Let \mathfrak{C}_2 be the collection of these copies of G_2 . Then, each element in \mathfrak{C}_2 has m'_2 mono-indexed edges. Therefore, the elements of \mathfrak{C}_2 contributes a total of $m'_1 m'_2$ mono-indexed edges.

It remains to determine the number of mono-indexed edges between G_1 and different copies of G_2 . The mono-indexed vertex of every non-mono-indexed edge of G_1 is adjacent to all vertices of the corresponding copy of G_2 , which is also i -uniform. The number of such mono-indexed edges is $(m_1 - m'_1)n_2$. Both end vertices of each mono-indexed edge of G are adjacent to n'_2 mono-indexed vertices of the corresponding copies of G_2 . The number of such mono-indexed edges is $2m'_1 n'_2$.

Therefore, the total number of mono-indexed edges in $G_1 \diamond G_2$ is $m'_1 + (m_1 - m'_1)m_2 + m'_1 m'_2 + (m_1 - m'_1)n_2 + 2m'_1 n'_2 = m'_1(1 + m'_2 + 2n'_2) + (m_1 - m'_1)(m_2 + n_2)$. \square

In view of Theorem 2.2, let us now proceed to discuss the sparing number of the edge corona of certain graphs. We shall first consider the edge corona of two path graphs.

Theorem 2.5. Let P_m and P_n be two paths on m and n vertices respectively, for $m, n > 1$. Then, the sparing number of the edge corona of P_m and P_n is

$$\varphi(P_m \diamond P_n) = \begin{cases} \frac{1}{2}m(n+2) - 1; & n \text{ is even} \\ \frac{1}{2}m(n+1) - 1; & n \text{ is odd.} \end{cases}$$

Proof. Let $G = P_m \diamond P_n$. Assume that an internal vertex v of P_m has a non-singleton set-label. Then, the $2 + 2n$ edges incident on v become non-mono-indexed. But, two copies of P_n whose vertices are adjacent to v become 1-uniform and $(n - 1)$ edges of each of these copies of P_n become mono-indexed. More over, $2n - 1$ edges between P_m and each of these two copies of P_2 become mono-indexed if n odd and $2n - 1$ edges between P_m and each of these two copies of P_2 become mono-indexed if n even. Therefore, In both cases, we have more mono-indexed edges than when G_1 is

1-uniform. Therefore, G has minimum number of mono-indexed edges when G_1 is 1-uniform.

If P_m is 1-uniform, each copy of P_n can be labeled in an injective manner alternately by non-singleton sets and singleton sets. Therefore, no edges in these copies need to be mono-indexed. Then, each vertex of P_m $\lfloor \frac{n}{2} \rfloor$ mono-indexed edges together with the mono-indexed vertices of the corresponding copy of P_n . Therefore, if n is even, G has $(m-1) + m \cdot \frac{n}{2} = \frac{1}{2}m(n+2) - 1$ mono-indexed edges and if n is odd, G has $(m-1) + m \cdot \frac{n-1}{2} = \frac{1}{2}m(n+1) - 1$ mono-indexed edges. \square

The sparing number of the edge corona of two graphs in which one is a path and the other is a cycle has been determined in the following theorems.

Theorem 2.6. *Let P_m be a path on m vertices and C_n be a cycle on n vertices, for $m > 1$. Then, the sparing number of the edge corona of P_m and C_n is*

$$\varphi(P_m \diamond C_n) = \begin{cases} \frac{1}{2}m(n+2) - 1; & n \text{ is even} \\ \frac{1}{2}m(n+5) - 2; & n \text{ is odd.} \end{cases}$$

Proof. Let $G = P_m \diamond C_n$. As proved in Theorem 2.5, G has minimum number of mono-indexed edges when P_m is 1-uniform. Then, each copy of C_n can be labeled in an injective manner alternately by singleton sets and non-singleton sets and hence no edges in these copies are mono-indexed. With respect to this labeling, each copy of C_n contains $\lceil \frac{n}{2} \rceil$ mono-indexed vertices and makes $\lceil \frac{n}{2} \rceil$ mono-indexed edges with each vertex of P_m . Therefore, if n is even, no copy of C_n need to have a mono-indexed edge and hence G has $(m-1) + m \cdot \frac{n}{2} = \frac{1}{2}m(n+2) - 1$ mono-indexed edges. If n is odd, then each copy of C_n must have a mono-indexed edge and hence G has $2(m-1) + m \cdot \frac{n+1}{2} = \frac{1}{2}m(n+5) - 2$ mono-indexed edges. \square

Theorem 2.7. *Let C_m be a cycle on m vertices and P_n be a path on n vertices, for $n > 1$. Then, the sparing number of the edge corona of C_m and P_n is*

$$\varphi(C_m \diamond P_n) = \begin{cases} \frac{1}{2}m(n+2); & n \text{ is even} \\ \frac{1}{2}m(n+1); & n \text{ is odd.} \end{cases}$$

Proof. Let $G = P_m \diamond C_n$. As we have already proved in Theorem 2.5, G has minimum number of mono-indexed edges when C_m is 1-uniform. Then, each copy of P_n can be labeled alternately by non-singleton sets and singleton sets and no edges in them are mono-indexed. Also, each copy of P_n contains $\lfloor \frac{n}{2} \rfloor$ mono-indexed vertices and makes the same number of mono-indexed edges with each vertex of C_m . Therefore, if n is even, G has $m + m \cdot \frac{n}{2} = \frac{1}{2}m(n+2)$ mono-indexed edges and if n is odd, G has $m + m \cdot \frac{n-1}{2} = \frac{1}{2}m(n+1)$ mono-indexed edges. \square

In the following result, we study the sparing number of the edge corona of two cycle graphs.

Theorem 2.8. *Let C_m and C_n be two cycles on m and n vertices respectively. Then, the sparing number of the edge corona of C_m and C_n is*

$$\varphi(C_m \diamond C_n) = \begin{cases} \frac{1}{2}m(n+2); & n \text{ is even} \\ \frac{1}{2}m(n+5); & n \text{ is odd.} \end{cases}$$

Proof. Let $G = P_m \diamond C_n$. Then, as we have stated in above theorems, G has minimum number of mono-indexed edges when C_m is 1-uniform and we can label each copy of C_n alternately by singleton sets and non-singleton sets. Hence, no edges in these copies will be mono-indexed. With respect to this labeling, each copy of C_n contains $\lceil \frac{n}{2} \rceil$ mono-indexed vertices and makes $\lceil \frac{n}{2} \rceil$ mono-indexed edges with each vertex of P_m . Therefore, if n is even, G has $m + m \cdot \frac{n}{2} = \frac{1}{2}m(n+2)$ mono-indexed edges. If n is odd, then each copy of C_n has one mono-indexed edge and hence G has $2m + m \cdot \frac{n+1}{2} = \frac{1}{2}m(n+5)$ mono-indexed edges. \square

So far, we have discussed about the edge corona of certain regular graphs having same vertex degree 2. Can we generalise this result to all regular graphs having same vertex degree? The following result provide a solution to this problem.

Theorem 2.9. *Let G_1 and G_2 be two r -regular graphs on m and n vertices respectively, for $m, n > 1$. Then, the sparing number of the edge corona of G_1 and G_2 is $m[n' + r(1 + \varphi_2)]$, where n' is the minimum number of mono-indexed vertices required in G_2 .*

Proof. Let $G = G_1 \diamond G_2$. Assume that an internal vertex v of G_1 has a non-singleton set-label. Then, the $r + 2n$ edges incident on v become non-mono-indexed. But, r copies of G_2 whose vertices are adjacent to v become 1-uniform and $rn - \varphi_2$ more edges of each of these copies of G_2 become mono-indexed, where φ_2 is the mono-indexed edges in G_2 . Let n_1 vertices having non-singleton set-labels in G_2 must be relabeled by singleton sets in these r copies of G_2 . Therefore, rn_1 edges between G_1 and each of these r copies of G_2 become mono-indexed. The total number of new mono-indexed edges in G is $r[rn - \varphi_2 + rn_1]$. Therefore, in this case, we have more mono-indexed edges than when G_1 is 1-uniform. Therefore, G has minimum number of mono-indexed edges when G_1 is 1-uniform.

If G_1 is 1-uniform, vertices of each copy of G_2 can be labeled in an injective manner alternately by distinct integral multiples of the set-labels of the corresponding vertices of G_2 . Then, the number of mono-indexed edges in each copy of G_2 is φ_2 . Then, the total number of mono-indexed edges in $G_1 \diamond G_2$ is $rm + rm\varphi_2 + mn' = m[n' + r(1 + \varphi_2)]$, where n' is the minimum number of mono-indexed vertices required in G_2 . \square

We can extend the above theorem for the edge corona of an r -regular graph G_1 and an s -regular graph G_2 , where $r < s$ as follows.

Theorem 2.10. *Let G_1 be an r -regular graph on m and n vertices and G_2 be an s -regular graph on n vertices, for $m, n > 1$ and $r \leq s$. Then, the sparing number of the edge corona of G_1 and G_2 is $m(n' + r(1 + \varphi_2))$, where n' is the minimum number of mono-indexed vertices required in G_2 .*

Proof. Let $G = G_1 \diamond G_2$. Assume that an internal vertex v of G_1 has a non-singleton set-label. Then, as mentioned in the previous theorem, $r + 2n$ edges incident on v become non-mono-indexed. But, r copies of G_2 corresponding to the edges incident on v become 1-uniform and hence $sn - \varphi_2$ more edges of each of these copies of G_2 become mono-indexed, where φ_2 is the mono-indexed edges in G_2 . Moreover, rn_1 edges between G_1 and each of these r copies of G_2 also become mono-indexed, where n_1 is the number of vertices having non-singleton set-labels in G_2 . Hence, the number of new mono-indexed edges in G is greater than the new non-mono-indexed edges in G . Therefore, in this case also, we have the minimum number of mono-indexed edges when G_1 is 1-uniform.

If G_1 is 1-uniform, vertices of each copy of G_2 can be labeled in an injective manner alternately by distinct integral multiples of the set-labels of the corresponding vertices of G_2 . Then, the number of mono-indexed edges in each copy of G_2 is varphi_2 . Hence, the total number of mono-indexed edges in G is $rm + m\varphi_2 + mn' = m[n' + r + \varphi_2]$, where n' is the minimum number of mono-indexed vertices required in G_2 . \square

Another important problem in this area is about the edge corona of two graphs in which one graph is a complete graph. First consider the edge corona of a path P_m and a complete graph K_n .

Theorem 2.11. *Let P_m be a path on m vertices and K_n be a complete graph on n vertices. Then, the sparing number of $P_m \diamond K_n$ is $\frac{1}{2}n(n+1)(m-1)$.*

Proof. Let $G = P_m \diamond K_n$. Then, G can be considered as the one point union of $m-1$ complete graphs on $n+2$ vertices. Therefore, by Theorem 1.6, each K_{n+2} has $\frac{1}{2}n(n+1)$ mono-indexed edges. Since each K_{n+2} are edge disjoint, by Theorem 1.5, the total number of mono-indexed edges is $\frac{1}{2}n(n+1)(m-1)$. \square

Next, let us consider the edge corona of a cycle C_m and a complete graph K_n .

Theorem 2.12. *Let C_m be a cycle on m vertices and K_n be a complete graph on n vertices. Then, the sparing number of $C_m \diamond K_n$ is $\frac{1}{2}mn(n+1)$.*

Proof. Let $G = C_m \diamond K_n$. Then, G can be considered as the one point union of m complete graphs K_{n+2} and by Theorem 1.6, each of these K_{n+2} has $\frac{1}{2}n(n+1)$ mono-indexed edges. Since each K_{n+2} are edge disjoint, by Theorem 1.5, the total number of mono-indexed edges is $\frac{1}{2}mn(n+1)$. \square

The above two results can be generalised for the edge corona of an r -regular graph G on m vertices and a complete graph K_n as follows.

Theorem 2.13. *Let G be an r -regular graph on m vertices and K_n be a complete graph on n vertices, where $r \leq n-1$. Then, the sparing number of $G \diamond K_n$ is $\frac{1}{4}rmn(n+1)$.*

Proof. Since G is an r -regular graph on m vertices, then the number of edges in G is $\frac{1}{2}rm$. Then, $G \diamond K_n$ can be considered as a one point union of $\frac{1}{2}rm$ complete graphs on $n+2$ vertices. Then, the total number of mono-indexed edges in $G \diamond K_n$ is $\frac{1}{2}rm \cdot \frac{1}{2}n(n+1) = \frac{1}{4}rmn(n+1)$. \square

3 Conclusion

In this paper, we have discussed about the sparing number of the edge corona of certain graphs. Some problems in this area are still open. For an r -regular graph G_1 and an s -regular graph G_2 , with $r > s$, estimation of the sparing number of their edge corona is very complex. For some values r and s , we get minimum number of mono-indexed edges when G_1 is 1-uniform and for some other values of r and s , we have minimum number of mono-indexed edges when G_1 is not 1-uniform. Hence, determining the sparing number of $G_1 \diamond G_2$ in such a situation still remains as an open problem.

In this paper, we have not addressed the problem of determining the sparing number of the edge corona of two graphs in which the first graph is a complete graph. The case when both the graphs are complete graphs are also not attempted.

For two arbitrary graphs, determining the sparing number of their edge corona is more complicated. The uncertainty in the adjacency and incidence pattern of arbitrary graphs makes this study complex. Hence, determining the sparing number of $G_1 \diamond G_2$ for arbitrary graphs G_1 and G_2 is also an open problem.

The problems related to verifying the admissibility of weak IASIs by other graph products of two arbitrary graphs and determining the corresponding sparing numbers are also open.

References

- [1] J. A. Bondy and U. S. R. Murty, **Graph Theory**, Springer, 2008.
- [2] K. P. Chithra, K. A. Germina and N. K. Sudev, *The Sparing Number of the Cartesian product of Certain Graphs*, Communications in Mathematics & Applications, **5**(1)(2014), 23-30.
- [3] K. P. Chithra, K. A. Germina and N. K. Sudev, *A Study on the Sparing Number of the Corona of Certain Graphs*, Research & Reviews: Discrete Mathematical Structures, **1**(2)(2014), 5-15.
- [4] R. Frucht and F. Harary, *On the Corona of Two Graphs*, Aequationes Math., **4**(3)(1970), 322-325.
- [5] J. A. Gallian, *A Dynamic Survey of Graph Labelling*, The Electronic Journal of Combinatorics, # DS 16, 2013.

- [6] K. A. Germina and T. M. K. Anandavally, *Integer Additive Set-Indexers of a Graph: Sum Square Graphs*, Journal of Combinatorics, Information and System Sciences, **37**(2-4)(2012), 345-358.
- [7] K. A. Germina and N K Sudev, *On Weakly Uniform Integer Additive Set-Indexers of Graphs*, International Mathematical Forum, **8**(37)(2013), 1827-1834.
- [8] R. Hammack, W. Imrich and S. Klavzar, **Handbook of Product graphs**, CRC Press, 2011.
- [9] F. Harary, **Graph Theory**, Addison-Wesley Publishing Company Inc., 1994.
- [10] Y. Hu and W. C. Shiu, *The spectrum of the edge corona of two graphs*, Electronic Journal of Linear Algebra, **20**(2010), 586-594.
- [11] N. K. Sudev and K. A. Germina, *A Characterisation of Weak Integer Additive Set-Indexers of Graphs*, Journal of Fuzzy Set Valued Analysis, **2014**(2014), Article Id: jfsva-0189, 7 pages.
- [12] N. K. Sudev and K. A. Germina, *Weak Integer Additive Set-Indexers of Graph Operations*, Global Journal of Mathematical Sciences: Theory and Practical, **6**(1)(2014), 25-36.
- [13] N. K. Sudev and K. A. Germina, *Further Studies on the Sparing Number of Graphs*, TechS Vidya e-Journal of Research, **2**(2013-14), 28-38.
- [14] N.K. Sudev and K. A. Germina, *A Note on Sparing Number of Graphs*, Advances and Applications in Discrete Mathematics, **14**(1)(2014), 51-65.
- [15] N.K. Sudev and K. A. Germina, *On Weak Integer Additive Set-Indexers of Certain Graph Classes*, Journal of Discrete Mathematical Sciences and Cryptography, to appear.
- [16] N. K. Sudev and K. A. Germina, *Weak integer Additive Set-Indexers of Certain Graph Products*, Journal of Informatics and Mathematical Sciences, **6**(1)(2014), 35-43.
- [17] D B West, (2001). *Introduction to Graph Theory*, Pearson Education Inc.